**Name:**

Fall 2020 Stat 311 Exam 2

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. A family has 3 children that are conveniently named A, B, and C. Is the event A is older than B independent from the event A is older than C? Explain. Compute the probability that A is older than B given that A is older than C. [10 pts]

2. It has been observed that an average of 3 crabs in a crab swarm are eaten every minute by their predators. Estimate the probability that 5 crabs will be eaten in the next 2 minutes [10 pts]
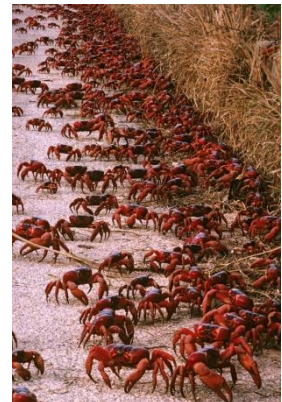
3.   In a country of 300 million people, 3 million are assumed to have contracted a scary illness. A test to determine whether or not a given patient has the disease is 99% reliable. If one wishes to know whether or not he or she is infected, what is the better strategy? (a) Go get tested? (b) Flip a coin? Explain.   [10 pts]

4.   Jimmy buys n Wonka chocolate bars, t of which contain a golden ticket. The wrappings are opened randomly without replacement. Let X be the number of Wonka bars opened until all the t golden tickets are found. Find the probability mass function of X and compute E[X].
                                                                                            [10 pts]

5.	100 students are randomly divided into pairs on day 1. They are randomly divided into pairs on day 2 again. Let X = 0, 2, 4,..., 100 be the number of students that had the same partner on both days. Compute $E[X]$. 	[10 pts]

6.	A blogger discovers two peculiar clubs. The Randomly Social Regressive and the Randomly Social Progressive clubs appear to be very different indeed!

	(a) The first club thinks it's the 1950s and goes by the motto "A man is a man and a woman is a woman". The blogger observes that there are 10 people waiting to enter the club, 6 of which are assigned the gender "woman" and 4 of which are assigned the gender "man". Thereupon 5 people are randomly admitted in . If X is the number of women that went in, compute P(X = k). 	[4 pts]

(b) The second club feels that people are randomly free to determine their gender once they cross the threshold to the club. The blogger observes that 10 people were waiting to enter. The first 5 are let in, thereupon 6 of the 10 randomly assume the gender "woman" and 4 randomly assume the gender "man". If Z is the number of women in this club, compute P(Z = k).   [4 pts]

(c) Are the random variables X and Z really any different? Explain.   [2 pts]

7. You play a game in which you may roll a die 1, 2, or 3 times. The choice whether to roll again or not is entirely yours. You get the last number that was rolled, in dollars, as your prize. Find the strategy that maximizes your expected payoff. Make sure to prove that this is indeed the best strategy and calculate the expected payoff if it is used. [Hint: Figure this out when only 2 rolls of the die are allowed first!]   [10 pts]

8. Using the weak law of large numbers, estimate the number of times a fair die must be tossed to insure that the ratio of outcomes resulting in a 6 to the total number of rolls is within 0.01 from 1/6, with probability 0.99 or higher.

[10 pts]

9. n words are randomly chosen from the set {mortal, coil, this, shuffle, off, to, be, not, or}. How many times do you expect the sentence "to be or not to be" to occur?   [10 pts]

10. A and B alternate rolling a pair of dice, stopping either when A rolls the sum 7 or when B rolls the sum 9. Assuming that A rolls first, find the probability that the final roll is made by A.

[10 pts]

Extra Credit

11. Lilli pond leafs labeled 0-10 form a bridge from one shore of the pond to the next. A frog is initially on leaf # 1 and a snake is waiting in ambush on leaf # 0. If the frog jumps to leaf # 0, it is dead, whereas, if it gets to leaf # 10, the frog survives. Given that the frog is on leaf # k, it will jump to leaf # k-1 with probability $\frac{k}{10}$ and it will jump to leaf # k+1 with probability $\frac{10-k}{10}$. For example, given that the frog is on leaf # 3, it will jump back to leaf # 2 with probability $\frac{3}{10}$ and it will jump forward to leaf number 4 with probability $\frac{10-3}{10} = \frac{7}{10}$. Find the probability that the frog survives.

0	1	2	3	4	5	6	7	8	9	10

[10 pts]

12. To enter a probability club, you must pay an entrance fee by playing one of the following games

   Game 1: Roll a fair die until the pattern 56 appears for the first time and pay a dollar for each time the die was rolled.

   Game 2: Roll a fair die until the pattern 66 appears for the first time and pay a dollar for each time the die was rolled.

   Is one game better than another? Calculate the expected price of each game.   [10 pts]